

MPHYCC-6

M.Sc. Sem II

Plasma Physics

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A transverse magnetohydrodynamic wave travelling in the direction of the magnetic field in a magnetized plasma. The velocity of such waves (the Alfven velocity or speed) is characteristic for a plasma of given properties.

A Alfven wave in a plasma is a low frequency travelling oscillation of the ions and the magnetic field. The ion mass density provides the inertia and the magnetic field line tension provides the restoring force.

The wave propagates in the direction of the magnetic field, although waves exist at oblique incidence and smoothly change into the magneto-sonic wave when the propagation is perpendicular to the magnetic field.

The motion of the ions and the perturbation of magnetic field are in the same direction and transverse to the direction of propagation.

### Alfven Velocity :-

The low frequency relative permittivity  $\epsilon$  of a magnetized plasma is given by

$$\epsilon = 1 + \frac{c^2 \mu_0 \rho}{B^2}$$

where  $B$  is the magnetic field strength,  $c$  is the speed of light,  $\mu_0$  is the permeability of the vacuum, and the mass density is

$$\rho = \sum_s n_s m_s$$



totalled over all charged Plasma Particles, each specie indexed by  $s$ , with ion-number density  $n_s$  and individual ionic mass  $m_s$ ; The Sun includes both electrons and ions.

The Phase velocity of an electromagnetic wave in such a medium is

$$v = \frac{c}{\sqrt{\epsilon}} = \frac{c}{\sqrt{1 + \frac{c^2 \mu_0 \rho}{B^2}}}$$

$$\approx v = \frac{v_A}{\sqrt{1 + \frac{v_A^2}{c^2}}} \quad \text{where } v_A = \frac{B}{\sqrt{\mu_0 \rho}}$$

is the Alfvén velocity if  $v_A \ll c$ , the  $v \approx v_A$ .  
 on the other hand, when  $v_A \rightarrow \infty$  then  $v \rightarrow c$ ,  
 i.e. at high field or low density, the velocity of the Alfvén wave approaches the speed of light, and the Alfvén wave becomes an ordinary electromagnetic wave.

Neglecting the contribution of the electrons to the mass density and assuming that there is a single ion species, we get.

$$v_A = \frac{B}{\sqrt{\mu_0 n_i m_i}} \quad \text{in SI}$$

$$v_A = \frac{B}{\sqrt{4\pi n_i m_i}} \quad \text{in Gauss}$$

$$v_A \approx (2.18 \times 10^{11} \text{ cm s}^{-1}) \left( \frac{m_i}{\text{mp}} \right)$$

when  $n_i$  is the ion number density and  $m_i$  is the ion mass.



In Plasma Physics, the Alfvén time  $\tau_A$  is an important time scale for wave phenomena. It is related to the Alfvén velocity by

$$\tau_A = \frac{a}{v_A}$$

where  $a$  denotes the characteristic scale of the system. For example,  $a$  could be the minor radius of the torus in a tokamak.

### Relativistic Case -

In 1993, Gradalin derived the Alfvén wave velocity using relativistic magnetohydrodynamics to be

$$v = \frac{c}{\sqrt{1 + \frac{(e+p)}{2P_m}}}$$

where  $e$  is the total energy density of plasma ~~the~~ particles,  $p$  is the total plasma pressure, and

$$P_m = \frac{B^2}{2\mu_0}$$

is the magnetic pressure. In the non-relativistic limit  $p \ll e \approx pc^2$ , and we immediately recover the expression ~~from~~